Physics with Maple

I.   Objective

To learn the basic algebra and calculus operations of Maple.

II.   Introduction

For the past two weeks we have been solving physics problems numerically. However, much of what we do in physics involves symbolic manipulation of mathematical formulas. For this reason, programs called **symbolic solvers** are useful. A symbolic solver is a program that is capable of performing symbolic mathematical operations. We will use a symbolic solver on Project Vincent called **Maple V**. To begin a Maple session type **add maple** at the Vincent prompt, and then type **xmaple** if you are in a **X-Windows** environment.

When possible, we must critically examine the results of a Maple session to ensure that it makes sense. Maple performs the mathematics so quickly that it is easy to fall into a pattern of automatically believing every result. It is best to approach the results critically, and *assume the result is wrong until proven correct.* We will discuss methods of checking solutions in the exercises.

Because Maple is best learned by example, the exercises contain a few examples of the commands you will learn today. For more examples and information about these commands, enter the help command by typing **?** or the Maple tutorial by typing **tutorial();** at the Maple prompt.

III.   Exercises

A.   Examples: Syntax, Algebra, and Graphs

Objective:	to use Maple commands to solve a physics problem
Where to begin:	start in Maple V on Project Vincent
What to do:	symbolically solve the mechanics problem described below. Follow the steps presented below.
What to turn in to your instructor:	nothing; this is an example for practice only
What to put in log book:	the time you begin your work, problems, solutions, new commands, etc.

(1) **Physics 221 Mechanics Problem:** We begin with a one dimensional collision to show Maple's ability to perform algebra. Suppose mass m moving initially with speed v_0 collides undergoes a one dimensional collision with a mass M initially at rest.
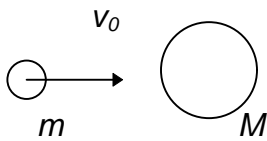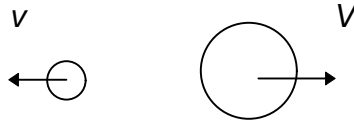
Before Collision   **After Collision**

We assume that the collision is elastic, meaning that the total kinetic energy of the system is conserved. We also know that the total momentum is conserved, and therefore one can write:

$$mv_0 = mv + MV$$

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}MV^2$$

Because we have two equations, we may solve the above system for any two unknowns. As an example, we will solve for v and V, and create a graph.

(2) **Basic Syntax, Line Termination, Assigning Variables:** We will now give Maple the above equations. To make our work easier, it is convenient to assign names to the equations, for example eq1 and eq2. To assign a variable name to a quantity use the symbol **:=**. Enter the equations in Maple as shown below:

    eq1:=m*v0=m*v+M*V;

    eq2:=1/2*m*v0^2=1/2*m*v^2+1/2*M*V^2;

Note the following punctuation symbols and their meanings:

Symbol	What it does
eq1:=	assigns the variable name **eq1** to the equation to the right of it (Note that the assign symbol may be used to assign a variable name to formulas, variables, lists, etc.)
;	terminates command lines: tells Maple to return the output
:	terminates command lines: tells Maple to suppress output (try it above and see)

Note also that Maple is case sensitive: it therefore distinguishes between upper and lower case letters.

(3) **Solving Sets of Simultaneous Equations:** Now we will solve for v and V. Enter the commands below:

    sol:=solve({eq1,eq2},{v,V});

The symbols are explained below:

Symbol	What it does
sol:=	assigns the name **sol** to the right-hand side of the equation
solve({eq},{var})	begins Maples routine for solving the system of equations (whose names are listed in the first set of curly brackets) for the specified variables (whose names are listed in the second set of curly brackets)
{eq1,eq2}	this is the list of equation names in our problem (note all lists in Maple are specified with curly brackets)
{v,V}	this is the list of variable names in our problem

Maple will return the output below:

$$sol:= \{v = v0, V = 0\}, \left\{v = \frac{(m-M)v0}{(m+M)}, V = \frac{2mv0}{(m+M)}\right\}$$

Because one equation was quadratic, there are two possible solutions, which Maple presents in two lists (curly brackets are for lists). The first solution corresponds to the time before the collision since the speeds are the same as the initial conditions, so we exclude it. The second solution is what we want.

(4) **Manipulating Solutions:**   We wish to choose the second solution set and to assign the variables *v* and *V* to the solutions. To do this we type:

assign(sol[2]);

Symbol	What it does
assign( )	assigns the variable names specified in the **solve** command to the solution found by the **solve** command
sol[2]	this specifies the second solution set in the solution list called **sol**

(5) **Checking Solutions:** Maple provides a way of checking solutions by substituting the solutions back into the original equation. Because the solving routine and checking routine are separate programs, it is highly unlikely that bugs exist in both that would cancel each other out. Therefore, this check is a meaningful way of gaining confidence in your solution. Execute the command below and note the result:

simplify(subs(sol,{eq1,eq2}));

$$\left\{\tfrac{1}{2}mv0^2 = \tfrac{1}{2}mv0^2, mv0 = mv0\right\}$$

The substitutions were made, and the result is a list of two identities. Since both sides of each equation are the same, our formulas really are solutions to the equations we specified.

Symbol	What it does
simplify( )	simplifies algebraic expressions
subs(sol,{eq1,eq2})	substitutes the solutions name **sol** into the system of equations {eq1,eq2}

(6) **Checking Limiting Cases:** Another way of checking our results is to check the formulas in the limits of its variables and compare the result with what you expect from physical intuition. For example, what do you expect the speed of mass m to become in the limit $M \to \infty$? To take this limit type:

limit(v,M=infinity);

What output does Maple give in the case $m=M$? the case $m \to \infty$? Are these results consistent with what you expect?

(7) **Making a 2D Plot:** Suppose now we wish to graph the behavior of v as a function of the mass ratio M/m. We first have to write the formula

$$v = \frac{m - M}{m + M} v_0$$

in a form that has no units. One way is to plot the ratio $y=v/v_0$ on the y-axis and the ratio $x=M/m$ on the x-axis. You can show that

$$y = \frac{1 - x}{1 + x}$$

To define y and then write M in terms of x in Maple type:

y:=v/v0:
M:=m*x:
y;

$$\frac{m - xm}{m + xm}$$

simplify(");

$$-\frac{-1 + x}{1 + x}$$

Symbol	What it does
simplify(")	simplifies the previous output. Here we could also use **simplify(y)** to get what we want

To make the plot, type:

plot(y,x=0..10);

Save your graph by going to the **File** menu on the graph window. Select the file type **Postscript** and give your file a name. To print out the graph type the following command at the vincent prompt: **lpr -Pdu139_lj4 -Dpostscript *filename*.** Your graph will be sent to the laser printer in room 139 Durham, and will be put in a bin corresponding to the first letter of your user name.

(8) **Save your work:** Use the pull down menus to save your **sess1.ms**. Once your session is saved, you may open it later using the **Open** command in the **File** menu.

B.   Application

Objective:	use Maple to solve a ballistic pendulum problem
Where to begin:	in Maple V (choose the command **New** under the **File** menu to begin a new session)
What to do:	obtain and check your solution using methods presented in the previous example
What to turn in to your instructor:	(1) a copy of your Maple session; (2) graph described below
What to put in log book:	the time you begin your work, problems, solutions, new commands, etc.

(1) **Ballistic Pendulum:** You have all worked this problem before. A ballistic pendulum is a large block of wood that is free to swing. A bullet is fired into the block, and by using the conservation of energy and linear momentum one may determine the initial speed of the bullet v_0 by measuring the maximum height h to which the block rises (see the figure below).

Before Collision   **After Collision**
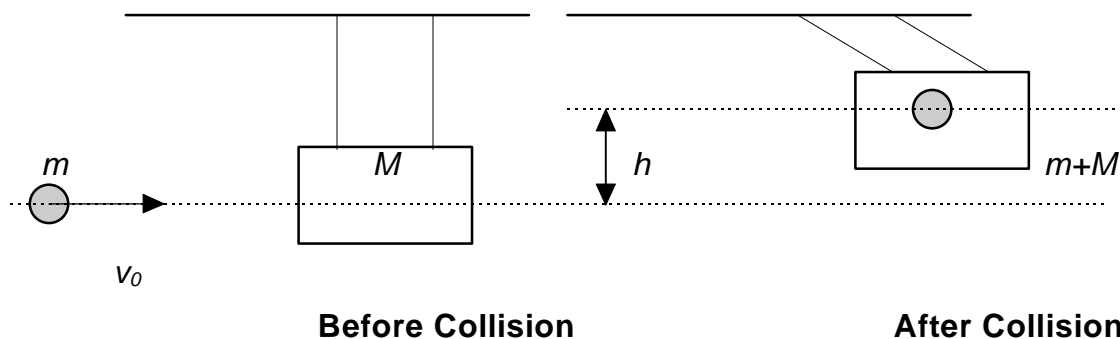
The conservation laws yield the equations

$$mv_0 = (m + M)V$$

$$\tfrac{1}{2}(m + M)V^2 = (m + M)gh$$

where V is the speed of the block immediately after the bullet is imbedded.

(2) **Solve the Equations:** Following the methods of the previous example, solve the system for V and h. Be sure check the solution using the **subs** command.

(3) **Check Limiting Cases:** Verify that the expressions you found for h and V are correct in the three limiting cases $m \to 0$, $M \to \yen$, $v_0 \to 0$.

(4) **Graph Results:** Generate a plot of your result for h (you will have to rescale it to make it unitless) versus the variable M/m over the range from 0 to 10.

(5) **Saving and Printing:** Save your session as **sess2.m** and print out both the text of the session and the graph to submit to your instructor.
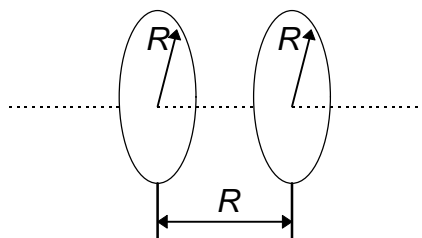
C.   Derivatives

Objective:	use Maple to solve a problem involving derivatives
Where to begin:	in Maple V (choose the command **New** under the **File** menu to begin a new session)
What to do:	use the differentiation command **diff** to solve the problem below
What to turn in to your instructor:	a copy of your Maple session
What to put in log book:	the time you begin your work, problems, solutions, new commands, etc.

(1) **Examples:** To differentiate an expression use the **diff** command. First assign the expression to a variable (see Exercise A), for example the variable **func**. Differentiation with respect to a variable is then done as shown below:

Maple Command	Mathematical Operation
diff(func,x);	$\dfrac{\partial}{\partial x}\,func$
diff(func,x$2);	$\dfrac{\partial^2}{\partial x^2}\,func$
diff(func,x$2,y);	$\dfrac{\partial}{\partial y}\,\dfrac{\partial^2}{\partial x^2}\,func$

(2) **Helmholtz Coils:** Helmholtz coils are a set of two circular coils of radius R whose centers are a distance R apart. A steady current I flows in the same direction around each coil as shown in the figure below.   The magnetic field *on the axis* at a distance x from the center of one coil is

$$B = \frac{m_0 I R^2}{2}\left[\frac{1}{(R^2 + x^2)^{3/2}} + \frac{1}{(2R^2 + x^2 - 2Rx)^{3/2}}\right]$$

(3) **Determine Derivatives:** Find the value of B, $\dfrac{dB}{dx}$, and $\dfrac{d^2B}{dx^2}$ at the midpoint between the two coils $x=R/2$ (note that the command **x:='x'** will clear the variable x of its previous values).

    D.   **Integrals**

Objective:	use Maple to solve a problem involving integrals
Where to begin:	in Maple V (choose the command **New** under the **File** menu to begin a new session)
What to do:	use the integration command **int** to solve the problem below
What to turn in to your instructor:	a copy of your Maple session
What to put in log book:	the time you begin your work, problems, solutions, new commands, etc.

(1) **Examples:** To integrate an expression use the **int** command.

Maple Command	Mathematical Operation
int(func,x);	$\int dxfunc$
int(int(func,x),y);	$\int dy \int dxfunc$
int(func,x=a..b);	$\int_a^b dxfunc$
int(int(func,x=a..b),y=c..d);	$\int_c^d dy \int_a^b dxfunc$

(2) **Problem:** The electric field due to a thin, uniformly charged, circular disk of charge Q and radius R at a point on the symmetry axis a distance z from the disk center can be written as

$$\vec{E} = \hat{z}\,\frac{zs}{4pe_0}\int_0^{2p} df \int_0^R \frac{rdr}{(z^2+r^2)^{3/2}}$$

where $s = Q/(pR^2)$ is the surface charge density. Evaluate the expression and obtain a formula for the magnitude of the electric field. Note that the symbol **E** is a forbidden variable name because Maple assigns it a special value. Also, the constant $\pi = 3.14...$ is specified in Maple by the variable **Pi** (first letter capitalized).-

(3) **Check a Limiting Case:** In the case that the disk becomes infinitely large, one expects that the electric field should approach that of a uniformly charged, infinite plane which is given by $E_{plane} = s/2e_0$. Verify this.

IV.   Appendix 1: Basic Maple Commands[1]

	Command	What it does
Help Command	?*topic*	gives help on a *topic*
Tutorial Command	tutorial( );	begins the Maple V tutorial session
Session Commands	restart;	clears all variables and begins a new session;
	save \`*sess.**m**\`	saves a Maple session to file *sess.**m*** (note that the *.**m*** extension is necessary)
	read \`*sess.**m**\`	reads a previously saved Maple session
Algebra Commands	simplify( );	simplifies algebraic expressions
	factor( );	factors algebraic expressions
	expand( );	expands algebraic expressions
	solve( );	solves specified system of equations for specified unknowns
	subs( );	substitutes specified variables into specified equations
	assign( );	assigns solutions to variable names
	evalf( );	numerically evaluates a function
Calculus Commands	diff( );	differentiates function
	int( );	integrates function
	limit( );	takes the limit of a function
	series( );	performs a series expansion of the function
Graphics Commands	plot( );	performs a 2D plot
	plot3d( );	performs a 3D plot

V.   Appendix 2: Saving and Printing in the X-Maple Environment

The table below gives instructions on how to perform simple operations when using **X-Maple**, the X-Windows version of Maple V.

How to Save Maple Session	pull down the **File** menu and select **Save**
How to Open Maple Session	pull down the **File** menu and select **Open**

How to Print Out a Maple Session	(1) pull down the **File** menu and select the **Print** option
	 (2) save the current session as a **Postscript** file
	 (2) use
	        lpr -P(printer name) **-Dpostscript** *filename*
	    command to print out file

How to Print Out a Graph	(1) in the graph window, pull down the **File** menu;
	 (2) choose the option **Print** and pick the format **Postscript**;
	 (3) use
	        lpr -P(printer name) **-Dpostscript** *filename*
	        from the vincent prompt to print the file

[1]Note that this table gives the command name, but not the syntax. For syntax examples see the section Exercises, or use the help command (type ?*command* at the Maple prompt).